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DECENTRALIZED ADAPTIVE CONTROL OF MANIPULATORS

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Abstract

This paper presents two new adaptive schemes for motion control of robot manipulators. The first controller possesses a partially decentralized structure in which the control input for each task variable is computed based on information concerning only that variable and on two "scaling factors" that depend on the other task variables. The need for these scaling factors is eliminated in the second controller by exploiting the underlying topology of the robot configuration space, and this refinement permits the development of a completely decentralized adaptive control strategy. The proposed controllers are computationally efficient, do not require knowledge of either the mathematical model or the parameter values of the robot dynamics, and are shown to be globally stable in the presence of bounded disturbances. Furthermore, the control strategies are general and can be implemented for either position regulation or trajectory tracking in joint-space or task-space. Computer simulation results are given for a PUMA 762 manipulator, and demonstrate that accurate and robust trajectory tracking is achievable using the proposed controllers. Experimental results are presented for a PUMA 560 manipulator and confirm that the proposed schemes provide simple and effective real-time controllers for accomplishing high performance trajectory tracking.

1. Introduction

Increasing the applicability, versatility, and reliability of robot manipulators requires the development of control systems capable of providing performance superior to that obtainable with conventional robot controllers. Of particular interest is designing control strategies that ensure accurate motion control in the presence of uncertainties in the robot dynamics and payload, external disturbances, and sensor noise. A promising approach to this problem is to use adaptive control methods. in which an attempt is made to compensate for uncertainties and disturbances by adjusting the controller parameters on-line based on the observed system performance. An advantage of the adaptive approach is the potential to continuously improve performance while the robot is executing the task.

Much of the research on adaptive robot control has focused on designing *centralized* control schemes, in which the control input for- each task variable depends on all of the other task variables. Many investigators have: considered this problem in recent years, and as a result of these studies two broad approaches to adaptive controller development have emerged. The first approach, called ***model- based adaptive control***, assumes that the structure of the manipulator dynamics is known but that the constant inertial parameters, which appear linearly in the dynamic model, are unknown [e.g., 1-5]. These controllers have been shown to be globally asymptotically stable and have performed well in computer simulations and experiments. However, implementation of these schemes with general multijointed manipulators is computationally intensive, and their design requires precise knowledge of the structure of the entire manipulator dynamic model, Furthermore, these adaptive controllers can lack robustness to unmodelled dynamics, sensor noise, and external disturbances [6,7]. In the second approach to adaptive motion control, referred to as ***performance-based adaptive control***, the adaptive laws adjust the controller gains directly based on the system performance. These schemes assume that very little information *is* available concerning the structure and the parameter values of the robot dynamic model [e.g, 8,9]. A disadvantage of this approach is that the controller derivations rely on the assumption that the adaptive elements in the controller can vary significantly more rapidly than the terms in the manipulator dynamics.. On the other hand, the performance-based methodology can be used to derive ***decentralized*** adaptive controllers, in which the control input for a particular task variable is computed based on information concerning only that variable. in contrast, adaptive schemes designed using the model-based approach cannot be implemented in decentralized form because in these schemes the robot dynamic model is incorporated directly into the control law, so that each control input depends on all of the system variables.

Decentralized adaptive schemes have been proposed for manipulators because of their computational simplicity, ease of implementation, robustness, and fault tolerance relative to centralized strategies. Studies of the decentralized adaptive motion control problem for rigid-link manipulators have indicated that the anticipated benefits of this approach to controller development can be realized in practice [e.g., 10-14]. Indeed, the controllers proposed in these investigations are extremely simple and computationally efficient, require very little model information, and have performed well in both computer simulations and experiments. However, the cent rol schemes [1 O- 14] are derived using a performance-based adaptive approach and rely on the assumption that the controller elements can vary significantly more rapidly than the terms in the manipulator dynamics. The goal of this paper is to introduce two new adaptive manipulator controllers that enjoy the simplicity, tommutational efficiency, robustness, and generality of the decentralized schemes presented in [10-14] and, at the same time, possess the attractive characteristic of unconditional global stability of the centralized adaptive strategies [I-5]. Very recently, there has been progress in this direction, with Fu [15] presenting a decentralized adaptive algorithm for robot tracking control that is globally stable. The control law given in [15] has the structure of a robust controller, however, and appears to generate excessive control action in order to provide good performance. The present paper proposes two new adaptive schemes for motion control of robot manipulators: a partially decentralized strategy and a completely, decentralized controller. The proposed algorithms are comput ationally efficient, do not require knowledge of either the mathematical model or the parameter values of the robot dynamics. and can be implemented for either position regulation or trajectory tracking in joint-space or task-space. It is shown that the control schemes are globally stable in the presence of bounded disturbances and that in the absence of disturbances the size of the residual position errors can be made arbitrarily small.

The paper is organized as follows. In Section 2, some preliminary facts are established and the overall structure of the proposed control schemes is given. The two new adaptive motion controllers are developed in Sections 3 and 4. The performance of the controllers is illustrated in Section 5 through a computer simulation study and in Section 6 through an experimental investigation. Finally, Section 7 summarizes the paper and draws some conclusions.

2. Problem Formulation

This paper considers the decentralized adaptive motion control problem for rigid-link manipulators. Let $\mathbf{y}$ define the position and orientation of the robot end-effector relative to a fixed user-defined reference frame and note that, in the most general case, the elements of $\mathbf{y}$ are local coordinates for some smooth manifold M of dimension m. The forward kinematic and differential kinematic maps between the robot joint coordinates $\theta \in \mathcal{N}$ and the end-effector coordinates $\mathbf{y}$ can be written as

$$\mathbf{y} = \mathbf{h}(\theta), \qquad \dot{\mathbf{y}} = J(\theta)\dot{\theta} \tag{1}$$

where $\mathcal{N}$ is a smooth manifold of dimension n, $\mathbf{h} : \mathcal{N} \to M$, and $J \in \Re^{m \times n}$ is the end-effector Jacobian matrix.

It is often desirable to formulate the manipulator control problem in terms of generaloperates directly in the space where the task is executed. A task-space formulation can also be realized if the manipulator is kinematically redundant (when $m < n$) by utilizing an "augmented task-space" approach [e.g., 16-20]. In what follows, we shall consider nonredundant and redundant robots together and formulate the manipulator control problem in terms of a set of n generalized coordinates x. Typically, x is obtained by augmenting y with a set of n -- m kinematic functions that define some auxiliary task to be performed by the manipulator. To retain generality, we shall require only that the elements of x are local coordinates for some smooth manifold $\mathcal{X}$ of dimension n, and that the kinematic relationship between θ and x is known and continuous and can be written in a form analogous to (1):

$$\mathbf{x} = \mathbf{h}_a(\theta), \qquad \dot{\mathbf{x}} = J_a(\theta)\dot{\theta} \tag{2}$$

where $\mathbf{h}_a : \mathcal{N} \rightarrow \mathcal{X}$ and $J_a \in \Re^{n \times n}$. Observe that for x to be a valid generalized coordinate vector, the elements of x must be independent in the region of interest; thus it will be assumed in our development that J_a *is* of full rank. Note also that joint-space control is trivially recovered by defining $\mathbf{h}_a$ to be the identity map.

Consider the manipulator dynamic model written in terms of the generalized coordinates x as

$$\mathbf{F} = H(\mathbf{x})\ddot{\mathbf{x}} + V_{cc}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{G}(\mathbf{x}) + \mathbf{d}(\mathbf{x}, \dot{\mathbf{x}}, t) \tag{3}$$

where $\mathbf{F} \in \Re^n$ is the generalized force associated with x, $H \in \Re^{n \times n}$ is the symmetric, positive-definite inertia matrix, $V_{cc} \in \Re^{n \times n}$ quantifies Coriolis and centripedal acceleration effects and is related to H via $\dot{H} = V_{cc} + V_{cc}^T$, and G $\in \Re^n$ *is* the vector of gravity forces. The term d $\in \Re^n$ is a vector of bounded but otherwise arbitrary disturbances that can represent unmodelled state-dependent effects (such as Coulomb friction) or time-dependent disturbances (such as the forces arising from a time-varying payload). The dynamics (3) represents a Hamiltonian system and therefore possesses a well-understood structure. For example, in addition to the properties already mentioned, it can be shown that the terms $H, \mathbf{G}$ are bounded functions of x whose time derivatives $\dot{H}, \dot{\mathbf{G}}$ are also bounded in x and depend linearly on x, and that the matrix V_{cc} is bounded in x and depends linearly on $\dot{x}$. These properties are established for any set of generalized coordinates x in [21,22] and will prove useful in the development of the proposed adaptive controllers.

Each of the control systems proposed in this paper consists of two subsystems: an adaptive scheme that produces the task-space control input **F** required to ensure that the system (3) evolves from its initial state to the desired final state along some specified trajectory $\mathbf{x}_d(t) \in \mathcal{X}$ (where $\mathbf{x}_d$ is bounded with bounded derivatives), and an algorithm for mapping the control input **F** to a physically realizable joint-space control torque $\mathbf{T} \in \Re^n$. It will be shown that these controllers also provide regulation if the final desired configuration $\mathbf{x}_d$ is specified as a constant and the intermediate trajectory is not stipulated. If **x** is chosen as described above, then the F → T map required by the control system is simply [e. g., 23]:

$$\mathbf{T} = J_a{}^T \mathbf{F} \tag{4}$$

However, when the manipulator is cinematically redundant then alternative definitions for x can lead to other **F** → T maps, and there is considerable flexibility associated with this approach to redundancy resolution [19,20,23,24]. For the remainder of the paper it is assumed that such a map can always be constructed and that, equivalently, the control input F can be commanded directly. Therefore the focus of the subsequent discussion is on the adaptive control of the system (3),

3. Partially Decentralized Adaptive Control Scheme

We now turn to the derivation of the first adaptive motion controller. Consider the following decentralized task-space control law structure:

$$F_i = a_i(t)\ddot{x}_{di} + b_i(t)\dot{x}_{di} + f_i(t) + k_{pi}(t)e_i + k_{vi}(t)\dot{e}_i \qquad \text{for } i = 1, 2, ..., n \tag{5}$$

where the subscript i refers to the ith element of the vector, $e_i = x_{di} - x_i$ is the ith component of the trajectory tracking error, and the scalars $a_i(t), b_i(t), f_i(t), k_{pi}(t)$, and

$k_{vi}(t)$ are adaptive gains whose update laws are to be determined. In this control law, the first three terms on the right hand side represent feedforward components and the last two quantities are feedback elements. For the subsequent development. it is notationally convenient to write the n components of the control law (5) together as follows:

$$\mathbf{F} = A(t)\ddot{\mathbf{x}}_d + B(t)\dot{\mathbf{x}}_d + \mathbf{f}(t) + K_p(t)\mathbf{e} + K_v(t)\dot{\mathbf{e}} \tag{6}$$

where $A = \text{diag}(a_i) \in \Re^{n\times n}$, $B = \text{diag}(b_i) \in \Re^{n\times n}$, $K_p = \text{diag}(k_{pi}) \in \Re^{n\times n}$, and $K_v = \text{diag}(k_{vi}) \in \Re^{n\times n}$. Applying the control law (6) to the manipulator dynamics (3) yields the tracking error dynamics:

$$H\ddot{\mathbf{e}} + V_{cc}\dot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{e} + \Phi_G + \Phi_H\ddot{\mathbf{x}}_d + \Phi_V\dot{\mathbf{x}}_d - \mathbf{d} = 0 \tag{7}$$

where $\Phi_G = \mathbf{f}-\mathbf{G}$, $\Phi_H = A-H$. and $\Phi_V = B-V_{cc}$. *The* adaptation laws for A, B, f, K_p, K_v are now derived using a Lyapunov-based design method. We specify that K_p and K_v possess both constant and adaptive components, so that $k_{pi}(t) = k_{p0i} + k_{pti}(t)$ and $k_{vi}(t) = k_{v0i} + k_{vti}(t)$ With $k_{p0i} > 0$ and $k_{v0i} > 0$ $\forall i$; note that this maybe viewed as simply requiring that k_{pi} and k_{vi} be set to positive values initially. Define the Lyapunov function candidate

$$\begin{aligned} V &= \frac{1}{2}\dot{\mathbf{e}}^T H\dot{\mathbf{e}} + \frac{1}{2}\mathbf{e}^T K_{p0}\mathbf{e} + \epsilon\dot{\mathbf{e}}^T H\mathbf{e}/(1 + \|\mathbf{e}\|) \\ &+ \frac{1}{2}c_1\mathbf{f}^T\mathbf{f} + \frac{1}{2}c_2\mathbf{a}^T\mathbf{a} + \frac{1}{2}c_3\mathbf{b}^T\mathbf{b} + \frac{1}{2}c_4\mathbf{k}_p^T\mathbf{k}_p + \frac{1}{2}c_5\mathbf{k}_v^T\mathbf{k}_v \end{aligned} \tag{8}$$

where ϵ and the c_i are positive scalar constants, $\|.\|$ denotes the standard Euclidean norm, $K_{p0} = \text{diag}(k_{p0i}) \in \Re^{n\times n}$, $\mathbf{a} = [a_1, ..., a_n]^T \in \Re^n$, $\mathbf{b} = [b_1, ..., b_n]^T \in \Re^n$, $\mathbf{k}_p = [k_{pt1}, ..., k_{ptn}]^T \in \Re^n$, and $\mathbf{k}_v = [k_{vt1}, ..., k_{vtn}]^T \in \Re^n$. Observe that V in (8) is a positive-definite, radially unbounded scalar function of e, ė, f, a, b, $\mathbf{k}_p$, and $\mathbf{k}_v$ provided ϵ is chosen so that

$$\epsilon < [\lambda_{min}(K_{p0})\lambda_{min}(H)]^{1/2}/\lambda_{max}(H) \tag{9}$$

where $\lambda_{min}(\cdot), \lambda_{max}(\cdot)$ denote the minimum and maximum eigenvalue of the matrix argument, respectively. Note that the Lyapunov function candidate (8) contains a "cross-product-term" $\dot{\mathbf{e}}^T H\mathbf{e}/(1+\|\mathbf{e}\|)$. Including such a term is proposed by Bayard and Wen [4] to provide a certain form of Lyapunov function derivative and by Koditschek [23] based on elegant arguments concerning the Riemannian geometry of the state-space of mechanical systems. Alternatively, this construction may be viewed as a natural extension of the use of general quadratic forms (instead of simple sums-of-squares) as Lyapunov functions for linear systems.

We now investigate the stability of the error dynamics (i) using the Lyapunov function candidate (8). Differentiating V along the vector field defined by (7) yields, after simplification

$$\begin{aligned}\dot{V} = &-\dot{\mathbf{e}}^T K_{v0}\dot{\mathbf{e}} - \frac{\epsilon}{1+\|\mathbf{e}\|}\mathbf{e}^T K_{p0}\mathbf{e} + \frac{\epsilon}{1+\|\mathbf{e}\|}\dot{\mathbf{e}}^T H\dot{\mathbf{e}} - \frac{\epsilon\dot{\mathbf{e}}^T\mathbf{e}}{\|\mathbf{e}\|(1+\|\mathbf{e}\|)^2}\dot{\mathbf{e}}^T H\mathbf{e}\\ &+\frac{\epsilon\mathbf{e}^T}{1+\|\mathbf{e}\|}(V_{cc}^T\dot{\mathbf{e}} - K_{v0}\dot{\mathbf{e}} + \mathbf{d}) + \dot{\mathbf{e}}^T\mathbf{d} + \mathbf{f}^T[c_1\dot{\mathbf{f}} - \mathbf{q}]\\ &+\mathbf{a}^T[c_2\dot{\mathbf{a}} - [\mathbf{q}\ddot{\mathbf{x}}_d]] + \mathbf{b}^T[c_3\dot{\mathbf{b}} - [\mathbf{q}\dot{\mathbf{x}}_d]] + \mathbf{k}_{pt}^T[c_4\dot{\mathbf{k}}_{pt} - [\mathbf{qe}]]\\ &+\mathbf{k}_{vt}^T[c_5\dot{\mathbf{k}}_{vt} - [\mathbf{q}\dot{\mathbf{e}}]] + \mathbf{q}^T[H\ddot{\mathbf{x}}_d + V_{cc}\dot{\mathbf{x}}_d + \mathbf{G}]\end{aligned} \tag{10}$$

where the identity $H = V_{cc} + V_{cc}^T$ is used. In (10), $K_{v0} = \mathrm{diag}(k_{v0i}) \in \Re^{n\times n}$, $\mathbf{q} = \dot{\mathbf{e}} + \epsilon\mathbf{e}/(1+\|\mathbf{e}\|)$ is a weighted and normalized position/velocity error, and $[\mathbf{zw}] = [z_1w_1, z_2w_2, \ldots, z_nw_n]^T \in \Re^n$ for any two n-vectors $\mathbf{z}$, $\mathbf{w}$. Examination of (10) suggests the following adaptation laws for f_i, a_i, b_i, k_{pti} and k_{vti}:

$$\begin{aligned}\dot{f}_i &= -\alpha_1 f_i + \frac{1}{c_1}q_i\\ \dot{a}_i &= -\alpha_2 a_i + \frac{1}{c_2}q_i\ddot{x}_{di}\\ \dot{b}_i &= -\alpha_3 b_i + \frac{1}{c_3}q_i\dot{x}_{di}\\ \dot{k}_{pti} &= -\alpha_4 k_{pti} + \frac{1}{c_4}q_i e_i\\ \dot{k}_{vti} &= -\alpha_5 k_{vti} + \frac{1}{c_5}q_i\dot{e}_i\end{aligned} \tag{11}$$

where the α_i are scalar functions of the form

$$\alpha_i = \alpha_{i0} + \alpha_{i1}\|\dot{\mathbf{e}}\|, \qquad \alpha_{i0} > 0, \alpha_{i1} > 0 \qquad i = 1,2,\ldots,5 \tag{12}$$

Note that in (11),(12) the σ-modification-like terms [26] are scaled by $\| \dot{e} \|$; this refinement will be shown to provide improved convergence properties in the common case in which the final desired manipulator configuration $\mathbf{x}_d$ is a constant. Substituting these adaptation laws into (10) yields

$$
\begin{aligned}
\dot{V} = & -\dot{\mathbf{e}}^T K_{v0} \dot{\mathbf{e}} - \frac{\epsilon}{1 + \| \mathbf{e} \|} \mathbf{e}^T K_{p0} \mathbf{e} + \frac{\epsilon}{1 + \| \mathbf{e} \|} \dot{\mathbf{e}}^T H \dot{\mathbf{e}} - \frac{\epsilon \dot{\mathbf{e}}^T \mathbf{e}}{\| \mathbf{e} \| (1 + \| \mathbf{e} \|)^2} \dot{\mathbf{e}}^T H \mathbf{e} \\
& + \frac{\epsilon \mathbf{e}^T}{1 + \| \mathbf{e} \|} (V_{cc}^T \dot{\mathbf{e}} - K_{v0} \dot{\mathbf{e}} + \mathbf{d}) + \dot{\mathbf{e}}^T \mathbf{d} - \alpha_1 c_1 \| \mathbf{f} \|^2 \\
& - \alpha_2 c_2 \| \mathbf{a} \|^2 - \alpha_3 c_3 \| \mathbf{b} \|^2 - \alpha_4 c_4 \| \mathbf{k}_{pt} \|^2 - \alpha_5 c_5 \| \mathbf{k}_{vt} \|^2 \\
& + \mathbf{q}^T [H \ddot{\mathbf{x}}_d + V_{cc} \dot{\mathbf{x}}_d + \mathbf{G}]
\end{aligned}
\tag{13}
$$

The properties of G, H, and V_{cc} established in [21,22] and summarized in Section 2 together with the boundedness of d, $\dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_d$ permit the following bound on V in (13) to be derived:

$$
\begin{aligned}
\dot{V} \leq & -\lambda_{min}(K_{v0}) \| \dot{\mathbf{e}} \|^2 - \frac{\epsilon}{1 + \| \mathbf{e} \|} \lambda_{min}(K_{p0}) \| \mathbf{e} \|^2 + \frac{\epsilon}{1 + \| \mathbf{e} \|} \lambda_{max}(H) \| \dot{\mathbf{e}} \|^2 \\
& + \frac{1}{4} \epsilon \lambda_{max}(H) \| \dot{\mathbf{e}} \|^2 + \epsilon k_v \| \dot{\mathbf{e}} \|^2 + v_{max} k_v \| \dot{\mathbf{e}} \|^2 - \alpha_{10} c_1 \| \mathbf{f} \|^2 \\
& - \alpha_{20} c_2 \| \mathbf{a} \|^2 - \alpha_{30} c_3 \| \mathbf{b} \|^2 - \alpha_{40} c_4 \| \mathbf{k}_{pt} \|^2 - \alpha_{50} c_5 \| \mathbf{k}_{vt} \|^2 \\
& + \eta_0 + \eta_1 \| \dot{\mathbf{e}} \| + d_{max} \| \dot{\mathbf{e}} \|
\end{aligned}
\tag{14}
$$

where η_0, η_1 are positive scalar constants obtained through routine manipulation, k_v is a scalar upper bound on the linear dependency of V_{cc} on x (i.e., $\| V_{cc} \|_F \leq k_v \| x \|$ Vx with $\| \cdot \|_F$ the Frobenius matrix norm), and d_{max} and v_{max} are scalar upper bounds on the (norms of the) disturbance vector d and desired velocity vector $\dot{\mathbf{x}}_d$, respectively.

An examination of the inequality (14) reveals that proper selection of the controller parameters K_{p0}, K_{v0}, and ϵ will ensure that the first six terms on the right side of (14) are negative-definite in e and e. Indeed, this desirable result is obtained provided these controller parameters are chosen so that

$$
\frac{1}{\epsilon} > \max \left\{ \frac{5\lambda_{max}(H) + 4k_v}{4\lambda_{min}(K_{v0}) - 4v_{max}k_v} , \frac{\lambda_{max}(H)}{[\lambda_{min}(K_{p0})\lambda_{min}(H)]^{1/2}} \right\}
\tag{15}
$$

where the inequality (9) is incorporated directly into this constraint for completeness. Note that the selection process for these parameters does not require knowledge of H, but instead only very conservative upper and lower bounds for this matrix. This is because the controller parameters appear in all terms in (15), and their selection can therefore be used to "mask" the (potentially considerable) uncertainty regarding H. Similar analysis of the remaining terms in (14) shows that f, A, B, K_{pt}, K_{vt} appear in only negative-definite quantities, and that the positive terms in this expression depend at most linearly on $\|\dot{\mathbf{e}}\|$. This implies that the set $\mathcal{S}$ defined as

$$\mathcal{S} = \{\mathbf{e}, \dot{\mathbf{e}}, \mathbf{f}, A, B, K_{pt}, K_{vt} \quad | \quad \dot{V} \geq 0\} \tag{16}$$

is compact and includes the origin. These properties of $\mathcal{S}$ together with the positive-definite, radially-unbounded structure of V in (8) indicate the existence of a constant value V^* of V such that $\dot{V} < 0$ whenever $V > V^*$, which is sufficient to ensure that e, $\dot{\mathbf{e}}$ and all of the adaptive gains are bounded [26].

Several observations can be made concerning the adaptive control strategy (5),(11). First note that the proposed control law is extremely simple, requires very little information concerning the manipulator dynamics, and is partially decentralized in that the control input for each task variable is computed based on information concerning only that variable and on two scaling factors $\|\mathbf{e}\|$ and $1+\|\mathbf{e}\|$. Thus the proposed control scheme provides a computationally efficient, modular, and readily implementable solution to the manipulator trajectory tracking problem. Next, consider the situation where there are no external disturbances, so that $\mathbf{d} \equiv 0$. Then, in the common case in which the final desired manipulator configuration $\mathbf{x}_d$ is a constant, the errors e, e can be made arbitrarily small by increasing the adaptation gains $1/c_i$; this result is established in the Appendix. Observe that this latter result indicates that the proposed scheme is well-suited for the position regulation problem. Note that increasing the adaptation gains does not ordinarily lead to large or rapidly varying manipulator actuator torques, since the $1/c_i$ govern the rate of adaptation of the controller gains and only indirectly influence the magnitudes of these terms [27]. This behavior is illustrated in the computer simulations and experiments in Sections 5 and 6. and represents an advantage of the adaptive control approach over strategies such as robust control, where high gains often lead to excessive control action. An additional observation is that the scalar adaptation gains $1/c_i$ can be replaced with the appropriate diagonal matrices for increased flexibility and performance; scalar gains are used in the above analysis for simplicity of development.

4. Completely Decentralized Adaptive Control Scheme

The adaptive control strategy (5),(11) is not completely decentralized because the control input for each task variable depends on the two scaling factors $\|e\|$ and $1+\|e\|$, which are functions of the tracking error for all other task variables. The need for these scaling factors is eliminated in this section by exploiting the topology of the robot configuration space, and this refinement permits the development of a completely decentralized control scheme. More specifically, observe that in deriving the controller (5),(11) the only assumption made regarding the manipulator configuration space is that the elements of the generalized coordinate vector x are local coordinates for some smooth manifold $\mathcal{X}$ of dimension n. In most cases, however, it is known that the configuration space is compact, and this additional feature can be important from the perspective of manipulator control. The idea of utilizing the topology of the manipulator configuration space in the design of control schemes was first proposed by Wen, Kreutz-Delgado, and Bayard [28]. These researchers present a class of controllers for all-revolute robots whose design took advantage of the fact that the manipulator joint-space is the n-torus $\mathcal{T}^n$; included in this class of controllers is a model-based adaptive strategy. Their approach essentially involves selecting configuration space error coordinates which reflect the topology of $\mathcal{T}^n$ and defining an "error potential energy" (or error metric) that is also compatible with this topology. In what follows, we revisit the manipulator control problem and consider the decentralized adaptive control of robots possessing a compact configuration space. For simplicity. we restrict our attention to joint-space control of all-revolute manipulators, and adopt definitions for configuration error and error potential energy 0117 " that are similar to the ones given in [2 S). We note, " however, that these ideas can be extended in a natural way to task-space control of all-revolute manipulators, and indeed to the control of manipulators with prismatic joints provided that these joints have finite travel.

Consider the following parameterization of configuration error $\mathbf{e} = \theta_d - \theta$ [28]:

$$\mathbf{q}_0 = [\cos(\frac{e_1}{2}), \cos(\frac{e_2}{2}), \ldots, \cos(\frac{e_n}{2})]^T$$
$$\mathbf{q}_1 = [\sin(\frac{e_1}{2}), \sin(\frac{e_2}{2}), \ldots, \sin(\frac{e_n}{2})]^T \qquad (17)$$

It is clear that q., $\mathbf{q}_1$ provide a reasonable parameterization for configuration error on 7" since $e_i = \mathrm{atan2}(q_{1i}, q_{0i})$ and $\mathbf{q}_1 = O$ implies c $= \mathbf{0}$ mod 2π; the relationship between this parameterization and the unit quaternion is obvious. The following expressions can be easily derived and will prove useful in the development of the proposed controller:

$$\dot{\mathbf{q}}_0 = -\frac{1}{2}[\mathbf{q}_1 \dot{\mathbf{e}}] \quad , \quad \dot{\mathbf{q}}_1 = \frac{1}{2}[\mathbf{q}_0 \dot{\mathbf{e}}]$$
$$\mathbf{q}_0^T \dot{\mathbf{q}}_0 + \mathbf{q}_1^T \dot{\mathbf{q}}_1 = 0 \quad , \quad \| \mathbf{q}_1 \| \leq n^{1/2} \qquad (18)$$

The terms $\mathbf{q}_0, \mathbf{q}_1$ permit the construction of an error potential energy (or error metric) that is compatible with the topology of the joint error space:

$$u = \mathbf{q}_1^T K_p \mathbf{q}_1 + (\mathbf{q}_0 - \mathbf{1})^T K_p (\mathbf{q}_0 - 1) \qquad (19)$$

where 1 $= [1, 1, \ldots, 1]^T \in \Re^n$ and $K_p = \mathrm{diag}(k_{pi}) > 0 \in \Re^{n \times n}$. The properties of error potential energies of this form are discussed in [28] and are not repeated here. Instead, it is simply mentioned that U is a smooth function of e and that the global minimum of u is O and occurs at e = O mod 4π.

With these preliminaries established, we now turn to the derivation of the second adaptive motion controller. Consider the decentralized joint-space control law structure

$$T_i = a_i(t)\ddot{\theta}_{di} + b_i(t)\dot{\theta}_{di} + f_i(t) + k_{pi}q_{1i} + k_{vi}\dot{e}_i \text{ for } i = 1, 2, \ldots, n \qquad (20)$$

where k_{pi}, k_{vi} are positive scalar constant feedback gains and $a_i(t), b_i(t), f_i(t)$ are scalar adaptive elements whose update laws are to be determined. Note that for simplicity of exposition, we are using constant" rather than adaptive feedback gains; adaptive gains can be derived utilizing an approach analogous to the one presented in Section 3. For the subsequent development, it is notationally convenient to write the n components of the control law (20) together as follows:

$$\mathbf{T} = A(t)\ddot{\theta}_d + B(t)\dot{\theta}_d + \mathbf{f}(t) + K_p\mathbf{q}_1 + K_v\dot{\mathbf{e}} \tag{21}$$

where $A = \text{diag}(a_i) \in \Re^{n\times n}$, $B = \text{diag}(b_i) \in \Re^{n\times n}$, $K_p = \text{diag}(k_{pi}) \in \Re^{n\times n}$, and $K_v = \text{diag}(k_{vi}) \in \Re^{n\times n}$. Applying the control law (20) to the joint-space analog of the manipulator dynamics (3) yields the tracking error dynamics:

$$H\ddot{\mathbf{e}} + V_{cc}\dot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{q}_1 + \Phi_G + \Phi_H\ddot{\theta}_d + \Phi_V\dot{\theta}_d - \mathbf{d} = 0 \tag{22}$$

where $\Phi_G = \mathbf{f} - \mathbf{G}$, $\Phi_H = A - H$, and $\Phi_V = B - V_{cc}$. The adaptation laws for A, B, $\mathbf{f}$ are now derived using a Lyapunov-based design method. The Lyapunov function candidate employed in this approach is constructed utilizing the error potential energy U defined in *(19):*

$$V = \frac{1}{2}\dot{\mathbf{e}}^T H\dot{\mathbf{e}} + \mathbf{q}_1^T K_p\mathbf{q}_1 + (q_0 - 1)^T K_p(q_0 - 1)$$
$$+ \epsilon\dot{\mathbf{e}}^T H\mathbf{q}_1 + \frac{1}{2}c_1\mathbf{f}^T\mathbf{f} + \frac{1}{2}c_2\mathbf{a}^T\mathbf{a} + \frac{1}{2}c_3\mathbf{b}^T\mathbf{b} \tag{23}$$

where all terms are defined as in Section 3. Observe that V in (23) is positive-definite provided ϵ is chosen so that

$$\epsilon < [2\lambda_{min}(K_p)\lambda_{min}(H)]^{1/2}/\lambda_{max}(H) \tag{24}$$

We now investigate the stability of the error dynamics (22) using the Lyapunov function candidate (23), Differentiating V *along* the vector field defined by (22) yields. after simplification

$$\dot{V} = -\dot{\mathbf{e}}^T K_v\dot{\mathbf{e}} - \epsilon\mathbf{q}_1^T K_p\mathbf{q}_1 + \frac{\epsilon}{2}\dot{\mathbf{e}}^T H[q_0\dot{\mathbf{e}}] + \epsilon\mathbf{q}_1^T(V_{cc}^T\dot{\mathbf{e}} - K_v\dot{\mathbf{e}}) + \mathbf{f}^T[c_1\dot{\mathbf{f}} - \mathbf{s}]$$
$$+ \mathbf{a}^T[c_2\dot{\mathbf{a}} - [\mathbf{s}\ddot{\theta}_d]] + \mathbf{b}^T[c_3\dot{\mathbf{b}} - [\mathbf{s}\dot{\theta}_d]] + \mathbf{s}^T[H\ddot{\theta}_d + V_{cc}\dot{\theta}_d + \mathbf{G} + \mathbf{d}] \tag{25}$$

where $s = \dot{\mathbf{e}} + \epsilon \mathbf{q}_1$ is a weighted position/velocity error. Examination of (25) suggests the following adaptation laws for f_i, a_i, b_i:

$$
\begin{aligned}
\dot{f}_i &= -\alpha_1 f_i + \frac{1}{c_1} s_i \\
\dot{a}_i &= -\alpha_2 a_i + \frac{1}{c_2} s_i \ddot{\theta}_{di} \\
\dot{b}_i &= -\alpha_3 b_i + \frac{1}{c_3} s_i \dot{\theta}_{di}
\end{aligned}
\tag{26}
$$

where the α_i are positive scalar constants. Substituting these adaptation laws into (25) yields

$$
\begin{aligned}
\dot{V} = &- \dot{\mathbf{e}}^T K_v \dot{\mathbf{e}} - \epsilon \mathbf{q}_1^T K_p \mathbf{q}_1 + \frac{\epsilon}{2} \dot{\mathbf{e}}^T H [\mathbf{q}_0 \dot{\mathbf{e}}] + \epsilon \mathbf{q}_1^T (V_{cc}^T \dot{\mathbf{e}} - K_v \dot{\mathbf{e}}) - \alpha_1 c_1 \| \mathbf{f} \|^2 \\
&- \alpha_2 c_2 \| \mathbf{a} \|^2 - \alpha_3 c_3 \| \mathbf{b} \|^2 + \mathbf{s}^T [H \ddot{\theta}_d + V_{cc} \dot{\theta}_d + \mathbf{G} + \mathbf{d}]
\end{aligned}
\tag{27}
$$

from which the following bound on V can be derived:

$$
\begin{aligned}
\dot{V} \leq &- \lambda_{min}(K_v) \| \dot{\mathbf{e}} \|^2 - \epsilon \lambda_{min}(K_p) \| \mathbf{q}_1 \|^2 + \frac{1}{2} \epsilon \lambda_{max}(H) \| \dot{\mathbf{e}} \|^2 \\
&+ n^{1/2} \epsilon k_v \| \dot{\mathbf{e}} \|^2 + v_{max} k_v \| \dot{\mathbf{e}} \|^2 - \alpha_1 c_1 \| \mathbf{f} \|^2 - \alpha_2 c_2 \| \mathbf{a} \|^2 \\
&- \alpha_3 c_3 \| \mathbf{b} \|^2 + \eta_0 + \eta_1 \| \dot{\mathbf{e}} \| + d_{max} \| \dot{\mathbf{e}} \|
\end{aligned}
\tag{28}
$$

where η_0, η_1 are positive scalar constants obtained through routine manipulation and v_{max} is a scalar upper bound on the (norm of the) desired velocity vector $\dot{\theta}_d$.

An examination of the inequality (28) reveals that proper selection of the controller parameters K_p, K_v, and ϵ will ensure that the first five terms on the right hand side of (28) are negative-definite in $\mathbf{q}_1$ and $\dot{\mathbf{e}}$. Indeed, this desirable result is obtained provided these controller parameters are chosen so that

$$
\frac{1}{\epsilon} > \max \left\{ \frac{\lambda_{max}(H) + 2n^{1/2} k_v}{2\lambda_{min}(K_v) - 2v_{max} k_v}, \ \frac{\lambda_{max}(H)}{[2\lambda_{min}(K_p)\lambda_{min}(H)]^{1/2}} \right\}
\tag{29}
$$

where the inequality (24) is incorporated directly into this constraint for completeness. Notice that, as in the previous section, the selection process for these parameters does not require knowledge of H, but rather only very conservative estimates for upper and lower bounds for this matrix. An analysis of the remaining terms in (28) shows that f. .-1. D appear in only negative-definite quantities, and that the positive terms in this expression depend at most linearly in $\|\dot{e}\|$. This implies that the set $\mathcal{S}$ defined as

$$\mathcal{S} = \{q_1, \dot{e}, f, A, B \mid \dot{V} \geq 0\} \tag{30}$$

is compact and includes the origin. As discussed in the previous section, these properties of $\mathcal{S}$ together with the structure of V in (23) are sufficient to ensure that q_p, $\dot{e}$ and all of the adaptive gains are bounded [26].

Several observations can be made concerning the adaptive control strategy (20),(26). First note that the proposed control law is extremely simple, requires very little information concerning the manipulator dynamics, and possesses a decentralized structure. Thus the strategy is computationally efficient, modular, and easy to implement. All of the observations made concerning the controller derived in Section 3 apply here without modification. For example, it can be shown that if there are no external disturbances and the final desired manipulator position is a constant, then the bounds on e,& can be made as small as desired.

5. Simulation Studies

The class of adaptive control schemes developed in Sections 3 and 4 is now applied to a large industrial robot through computer simulation. The robot chosen for this simulation study is the six degree-of-freedon (DOF) PUMA 762 manipulator. The simulation environment incorporates models of all important dynamic subsystems and phenomena, such as the full nonlinear arm dynamics, joint stiction, sensor noise, and transmission effects, and therefore provides the basis for a realistic evaluation of controller performance [29]. The control laws are applied to the manipulator model with a sampling period of" two milliseconds, and all integrations required by the controller are implemented using a simple trapezoidal integration rule with a time-step of two milliseconds. The PUMA

762 manipulator possesses a conventional design with six revolute joints configured in a "waist-shoulder-elbow-wrist" arrangement, as shown in Figure 1. Precise values for all kinematic. and dynamic model parameters for this robot are given in [30] and are used in the simulation of the arm kinematics and dynamics. A brief characterization of the overall robot dimensions can be made by noting that the reach of the arm is approximately 50in and the total weight is almost 850lb. The simulation study consists of two parts: a nominal *performance study in which no* external disturbances are applied to the robotic system. and a ***robustness study*** where a payload-related disturbance is introduced. It is noted that throughout the computer simulation studies, the unit of length is inch, the unit of time is second, the unit of angle is degree, and the unit of force is pound, unless stated otherwise,

The first simulation study demonstrates the capability of the control scheme (5),(11) to provide accurate task-space trajectory tracking under nominal conditions, in which no external disturbances arc present. The PUMA robot is initially at rest with the joint-space position $0(0) = [0, 90, 180, 0, -90, 0]^T$. The robot is commanded to smoothly move its end-effector 20in in the z-direction, 20in in the y-direction, and 20in in the z-direction in 2 seconds according to the following temporal trajectories:

$$x(t) = x_i + 10(1 - \cos(\pi/2)t)$$

$$y(t) = y_i + 10(1 - \cos(\pi/2)t)$$

$$z(t) = z_i + 10(1 - \cos(\pi/2)t)$$

for $t \in [0, 2]$, where $[x_i, y_i, z_i]$ are the initial position coordinates of the end-effecter and all motions are specified relative to the base coordinate frame. Additionally, the robot is required to smoothly change its end-effecter orientation from an initial orientation matrix R_{init} to a final orientation matrix R_{final}, where

$$R_{init} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad R_{final} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and each rotation matrix R is specified relative to the base frame of the robot. The manipulator dynamics arid the final desired manipulator position is a constant, then the bounds on $\mathbf{e}, \dot{\mathbf{e}}$ can be made as small as desired. For simplicity of development, we present the analysis for the adaptive strategy (20),(26), and note that the results obtained hold for both of the controllers proposed in this papa-.

Proposition: Consider the joint-space analog of the manipulator dynamics (3) with no external disturbances

$$T = H(\theta)\ddot{\theta} + V_{cc}(\theta, \dot{\theta})\dot{\theta} + G(\theta) \tag{.41}$$

and the decentralized adaptive controller (20),(26) for the case in which the desired final position θ_d is a constant (here $\alpha_1 = \alpha$ and $1/c_1 = \beta$ for notational convenience):

$$\begin{aligned} T_i &= f_i(t) + k_{pi}q_{1i} + k_{vi}\dot{e}_i \\ \dot{f}_i &= -\alpha f_i + \beta s_i \\ s_i &= \dot{e}_i + \epsilon q_{1i} \\ \alpha &= \alpha_0 + \alpha_1 \| \dot{\mathbf{e}} \| \end{aligned} \tag{.42}$$

The controller (A2) ensures that the manipulator dynamics (Al) evolves so that $\mathbf{q}_1$, e, f arc globally uniformly ultimately bounded provided that

$$\frac{1}{\epsilon} > \max \left[\frac{\lambda_{max}(H)}{(2\lambda_{min}(K_p)\lambda_{min}(H))^{1/2}} \quad , \quad \frac{\lambda_{max}(H) + 2n^{1/2}k_v}{2\lambda_{min}(K_v)} + \frac{(\lambda_{max}(K_v))^2}{4\lambda_{min}(K_p)\lambda_{min}(K_v)} \right] \tag{A3}$$

Moreover, the ultimate bounds on $\mathbf{q}_1$, e can be made arbitrarily small.

Proof: Applying the control law (A2) to the manipulator dynamics (Al) yields the closed-loop dynamics:

$$He + V_{cc}\dot{\mathbf{e}} + K_v\dot{\mathbf{e}} + K_p\mathbf{q}_1 + \Phi_G = 0 \tag{.44}$$

Consider the Lyapunov function candidate

$$\begin{aligned} V = \frac{1}{2}\dot{\mathbf{e}}^T H\dot{\mathbf{e}} + \mathbf{q}_1^T K_p \mathbf{q}_1 + (\mathbf{q}_0 - 1)^T K_p(\mathbf{q}_0 - 1) \\ + \epsilon\dot{\mathbf{e}}^T H\mathbf{q}_1 + \frac{1}{2\beta}\Phi_G{}^T\Phi_G \end{aligned} \tag{.45}$$

temporal trajectory for this orientation change is specified using the standard "equivalent-angle-axis" parametrization of $SO(3)$ given by $\phi\hat{k} \in \Re^3$. where $\hat{k}$ is a unit vector aligned with the axis of rotation from the initial to the final desired orientation and ϕ is the angle of rotation about this axis. In this simulation. the desired orientation trajectory. is $\phi_d = 75°(1 - \cos(\pi/2)t)$ and $\hat{k}_d = [0, 0, 1]^T$ for $t \in [0, 2]$ [e.g., 31]. Thus the robot is commanded to translate its end-effector approximately 35in and rotate its end-effector through 150° in 2 seconds. The decentralized task-space control strategy (5),(11) is utilized to achieve the specified trajectory tracking. The adaptive gains A, B, f are set to zero initially, while K_p, K_v are initialized to $K_{p0} = K_{v0} = \text{diag}(20 \quad 20 \quad 20 \quad 1 \quad 1 \quad 1)$. The remaining controller parameters are set as follows: $\epsilon = 10$ and $\alpha_{i0} = \alpha_{i1} = 0.001$, $1/c_i = 10$ for $i = 1,2,\ldots,5$. The results of this simulation arc given in Figures 2a-2c, and indicate that accurate task-space trajectory tracking is achieved.

The next simulation in this study investigates the robustness of the controller (5),(11) to external disturbances. Note that when the manipulator payload mass m is time-varying, the skew-symmetry of the matrix $\dot{H} - 2V_{cc}$ is disrupted [32]. This result is important since most adaptive trajectory tracking schemes (including the ones proposed in this paper) exploit this skew-symmetry in their development. As a consequence, when the payload varies with time and skew-symmetry is lost, then the 'extra" terms in the manipulator dynamic model which arise from the time-variation of the payload represent an external disturbance to the nominal model. To examine the effect of this disturbance, the controller (5),(11) is utilized to repeat the task specified in the first simulation, but here a time-varying payload is introduced with mass given by $m(t) = 25(1 + \cos\frac{\pi}{2}t)$. Thus the payload mass smoothly decreases from 50lb to 0lb during the trajectory. In this simulation, all of the controller parameters are set to the values used in the nominal study described above. The results of this simulation are given in Figures 3a-3c, and indicate that accurate and robust task-space trajectory tracking is achieved despite the gross and continuous variation in payload mass.

6. Experimental Studies

The class of robot control schemes developed in Sections 3 and 4 is now applied to an industrial robot in a series of experiments. The experimented facility utilized fox this study is the J PL Robotics Research Laboratory and consists of a Unimation PUMA 560 arm and controller, and a DEC MicroVAX 11 computer; a functional diagram of this testbed is given in Figure 4. The MicroVAX 11 hosts the RCCL (Robot Control "C" Library) software, which was originally developed at Purdue University [33] and subsequently modified and implemented at JPL [34]. During the operation of the arm, a hardware clock constantly interrupts the 1/0 program resident in the Unimation controller at a preselected sampling period T_s, which can be chosen as 7, 14, 28, or 54 milliseconds; for all experiments reported here $T_s =$ 7ms. At every interrupt, the 1/0 program gathers information about the state of the arm (such as joint encoder readings), and interrupts the control program in the MicroVAX 11 to transmit this data. The 1/0 program then waits for the control program to issue a new set of control signals, and then dispatches these signals to the appropriate joint motors. Therefore, the MicroVAX 11 acts as a digital controller for the PUMA arm and the Unimation controller is effectively bypassed and is utilized merely as an 1/0 device to interface the MicroVAX II to the joint motors. A complete description of this testbed is beyond the scope of the paper; the interested reader is referred to [34].

The decentralized joint-space control scheme (20),(26) is selected for implementation in the present experimental study. The experiment consists of a nominal performance study and a robustness study, and in this way parallels the computer simulation investigation in Section 5. In the nominal performance study, the scheme (20),(26) is implemented for trajectory tracking control of the PUMA arm waist joint θ_1 while the other joints are held steady using the Unimation controller. In contrast, in the robustness study, the other joints arc given a large motion using the Unimation controller while θ_1 tracks a trajectory similar to the one specified in the nominal study. Observe that this motion of the joints distal to the waist joint has the effect of a time-varying payload. Recall from the previous section that this results "in the application of a time-varying external disturbance to the nominal robot dynamics.

In the nominal performance test, the PUMA arm is initially at rest with joint-space position $\theta(0) = 0$. The waist joint is commanded to change smoothly from the initial position $\theta_1 = 0$ to the goal position $\theta_1 = 90°$ in 2 seconds using the cycloidal trajectory $\theta_{d_1} = 45°(t - \sin \pi t)$ for $t \in [0, 2]$. The control law (20).(26) is slightly modified to include the capacity for adaptation in the feedback gains using an approach similar to the one utilized in deriving (11); this modified scheme is then used to achieve the desired trajectory tracking. The adaptive gains A, B, f, K_p, K_v are initialized to zero and the remaining controller parameters are set to the following values: $\epsilon = 1$, $1/c_1 = 1/c_2 = 0$, $1/c_3 = 0.1$, $1/c_4 = 0.2$, $1/c_5 = 0.5$, and $\alpha_i = 0$ for $i = 1,2,\ldots,5$. Note that, in view of the modest computing power available and the relatively high communication overhead in the testbed [34], no feedforward elements are used in the experiments. While the arm is in motion, the reading of the waist joint encoder at each sampling instant is recorded directly from the arm, converted into degrees and stored in a data file. Figure 5a shows the desired and actual trajectories of the waist joint angle, and the tracking error is shown in Figure 5b. It is seen that the joint angle $\theta_1(t)$ tracks the desired trajectory $\theta_{d1}(t)$ very closely, and the peak value of the tracking error $e(t) = \theta_{d1}(t) - \theta_1(t)$ is 1.40°, The initial lag in the θ_1 response is due to the large stiction (static friction) present in the waist joint. Figures 6a and 6b show the tracking performance of the waist joint for the same motion using the Unimation controller, which is operating with a sampling period of 1 millisecond; it is seen that the peak joint tracking error in Figure 6b is 5.36°. By comparing Figures 5b and 6b, it is evident that the tracking performance of the adaptive controller is noticeably superior to that of the Unimation controller, despite the fact that the Unimation control loop is seven times faster than the adaptive control loop.

The next experiment investigates the performance of the proposed controller when additional robustness is required for successful completion of the desired task. In this experiment, the control law (20),(26) is utilized to track a desired waist joint trajectory similar to the one specified in the first experiment. and the arm is initially in the same configuration. In this case, however, the Unimation controller is used to smoothly move the shoulder joint θ_2 and elbow joint θ_3 from the initial zero positions to $\theta_2 = -90°$ and $\theta_3 = 90°$ in 3 seconds during the execution of the task. observe that this is a gross and rapid variation in the arm configuration. which produces a large and fast variation in the arm inertia as seen at the waist joint. As indicated in Section 5, this variation presents a large external disturbance to the nominal waist joint dynamics. The desired waist joint trajectory for this experiment is $\theta_{d_1} = 20°(t - \sin(2\pi t/3))$ for $t \in [0,3]$, and all adaptive gains and controller parameters are set to the values used in the first experiment. The results of this experiment are shown in Figures 7a and 7b, and indicate that accurate and robust trajectory tracking is achieved despite the large external disturbance.

7. Conclusions

Two decentralized adaptive control schemes for robot manipulators are presented in this paper. The simplicity of the proposed controllers leads to computational efficiency, and makes the schemes particularly suitable for implementation as real-time control algorithms with a high sampling rate. The capabilities of the proposed control schemes are illustrated through a computer simulation study and an experimental investigation. The results of these studies demonstrate that the controllers provide a simple and effective means of obtaining high performance trajectory tracking. Future work will focus on the application of the proposed approach of controller development to the problem of robot compliant motion control.

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and observe that V is a fiositive-definite, radially-unbounded function of $q_1, \dot{e}, \Phi_G$ provided the controller parameters are chosen so that (A3) is satisfied. Differentiating (A5) along (.44) and simplifying gives

$$\dot{V} = -\dot{e}^T K_v \dot{e} - \epsilon q_1^T K_p q_1 + \frac{\epsilon}{2}\dot{e}^T H[q_0 \dot{e}] + \epsilon q_1^T (V_{cc}^T \dot{e} - K_v \dot{e}) + \frac{1}{\beta}\Phi_G^T (\dot{\Phi}_G - 3s)$$

$$= -\dot{e}^T K_v \dot{e} - \epsilon q_1^T K_p q_1 + \frac{\epsilon}{2}\dot{e}^T H[q_0 \dot{e}] + \epsilon q_1^T (V_{cc}^T \dot{e} - K_v \dot{e})$$

$$- \frac{\alpha}{\beta} \| \Phi_G + \frac{1}{2\alpha}(\dot{G} + \alpha G) \|^2 + \frac{1}{4\alpha\beta} \| \dot{G} + \alpha G \|^2 \tag{A6}$$

The properties of G established in [21,22] and summarized in Section 2 permit the following bounds to be derived:

$$\frac{1}{4\alpha\beta} \| \dot{G} + \alpha G \|^2 \leq \frac{1}{\beta}[\gamma_1 + \gamma_2 \| \dot{e} \|] \tag{A7}$$

$$- \frac{\alpha}{\beta} \| \Phi_G + \frac{1}{2\alpha}(\dot{G} + \alpha G) \|^2 \leq -\frac{1}{\beta}(\alpha_0 + \alpha_1 \| \dot{e} \|) \| \Phi_G + W_G \|^2$$

$$\leq -\frac{\alpha_0}{\beta} \| \Phi_G \|^2 + \frac{1}{\beta}(\gamma_3 + \gamma_4 \| \Phi_G \|)$$

where the γ_i are positive seal ar constants obtained through routine manipulation and W_G is a bounded function of q_1, $\dot{e}$. The inequalities (A7) then permit V in (A6) to be bounded as follows:

$$\dot{V} \leq -[\| q_1 \| \; \| \dot{e} \|] Q \begin{bmatrix} \| q_1 \| \\ \| \dot{e} \| \end{bmatrix} - \frac{\alpha_0}{\beta} \| \Phi_G \|^2 + \frac{1}{\beta}(\gamma_1 + \gamma_2 \| \dot{e} \|) + \frac{1}{\beta}(\gamma_3 + \gamma_4 \| \Phi_G \|) \tag{A8}$$

where the matrix Q is defined as

$$Q = \begin{bmatrix} \epsilon\lambda_{min}(K_p) & -\frac{1}{2}\epsilon\lambda_{max}(K_v) \\ -\frac{1}{2}\epsilon\lambda_{max}(K_v) & \lambda_{min}(K_v) - \frac{1}{2}\epsilon\lambda_{max}(H) - n^{1/2}\epsilon k_v \end{bmatrix}$$

and is positive-definite provided the controller parameters are chosen as in (A3).

Note that if $f : [0, \infty) \rightarrow \Re$ is defined as $f(x) = (a_0 + a_1 x)/a - a_2 x^2$ with a, a_0, a_1, a_2 positive scalar constants then there exist positive scalar constants b_0, b_2 such that $g(x) = b_0/a - b_2 x^2 > f(x) \, \forall x \in [0, \infty)$ (for example, choose $b_2 = a_2/4$ and $b_0 = a_0 + a_1^2/a_2 a$). This result allows the following upper bound on V in (A8) to be established:

$$\dot{V} \leq -\frac{1}{4} z^T Q z - \frac{\alpha_0}{4\beta} \| \Phi_G \|^2 + \frac{1}{\beta} B \tag{A9}$$

where $z = [\| q_1 \| \| \dot{e} \|]^T$ and B is a positive scalar constant that dots not increase when β is increased. Examination of (A5),(A9) reveals that there exist positive scalar constants λ_i independent of β such that the following bounds on $V, \dot{V}$ can be derived:

$$\lambda_1 \| z \|^2 + \frac{1}{2\beta} \| \Phi_G \|^2 \leq V \leq \lambda_2 \| z \|^2 + \frac{1}{2\beta} \| \Phi_G \|^2$$

$$\dot{V} \leq -\lambda_3 \| z \|^2 - \frac{\lambda_4}{\beta} \| \Phi_G \|^2 + \frac{1}{\beta} B \qquad \text{(Ale)}$$

Straightforward application of the global uniform ultimate boundedness theorem of Corless and Leitmann [35] to (A1O) yields the following ultimate bounds for $\| z \|, \| \Phi_G \|$ (see [35] for a discussion of ultimate boundedness):

$$\| z \|^2 \leq \frac{1}{\beta} \left(\frac{\lambda_2 B}{\lambda_1 \lambda_3} + \frac{B}{2\lambda_1 \lambda_4} \right)$$

$$\| \Phi_G \|^2 \leq \frac{2\lambda_2 B}{\lambda_3} + \frac{B}{\lambda_4}$$

Thus $\| z \|, \| \Phi_G \|$ are ultimately bounded, which implies that $q_1, \dot{e}$, f are ultimately bounded, Moreover, the ultimate bound on $\| z$ l!?== $\| q_1 \|^2 + \| \dot{e} \|^2$ can be decreased as desired simply by increasing β.

It is interesting to note that the results presented in this Appendix are actually more general than indicated by the statement of the proposition, For example, if there are external disturbances d present such that d is bounded in x and d is bounded in x and grows at most linearly with x, then arbitrarily small regulation errors can again be concluded. This is easily verified by combining d with G in the preceding analysis and observing that all of the arguments and conclusions still remain valid.

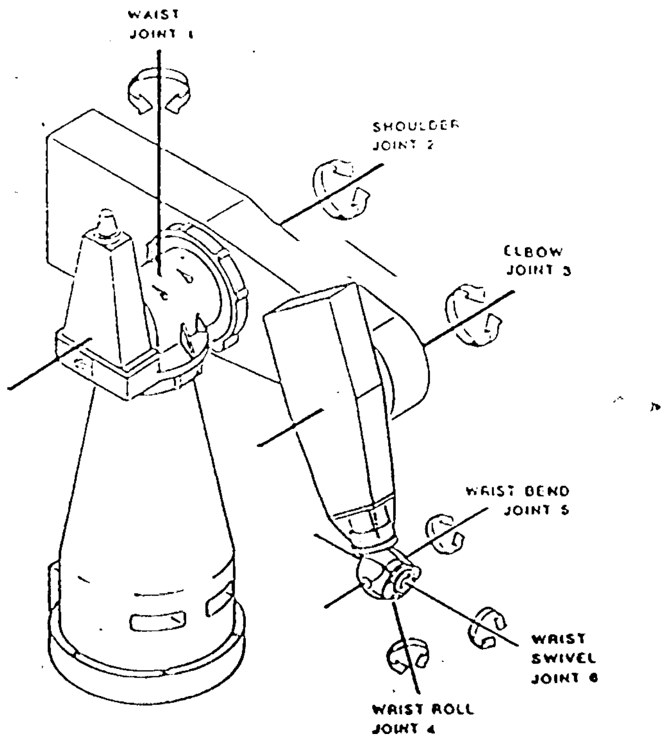

Figure 1: Illustration of PUMA 762 robot

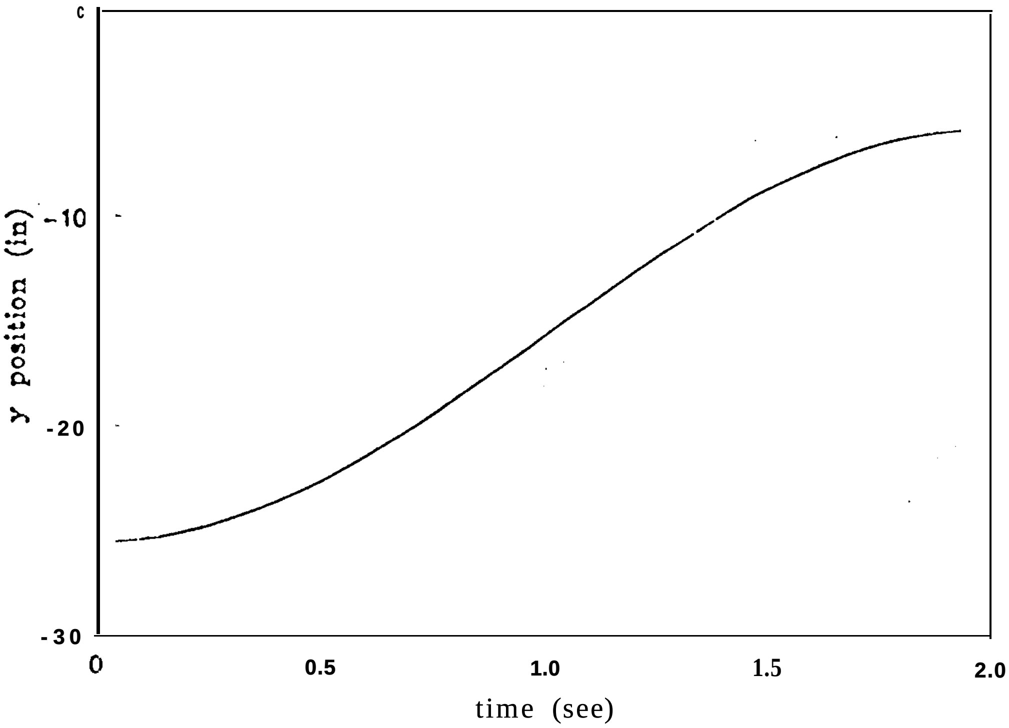

Figure 2a: Desired (dashed) and actual (solid) response of end-effecter y coordinate in PUMA arm simulation (nominal case)

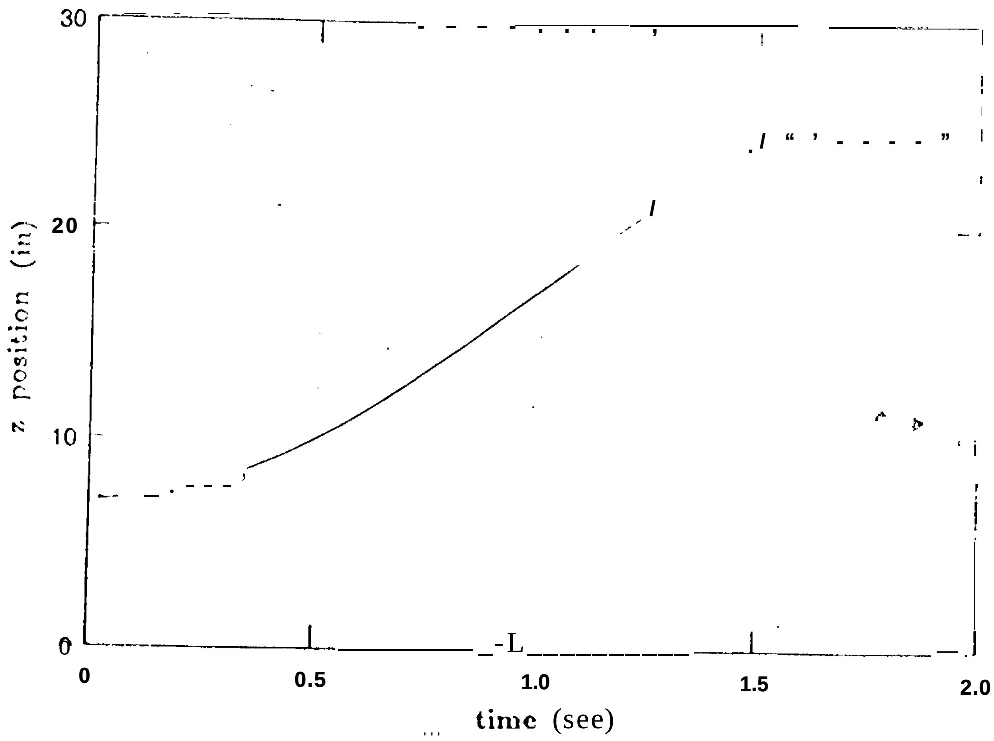

Figure **2b:** Desired (dashed) and actual (solid) response of end-effecter z coordinate in PUMA arm simulation (nominal case)

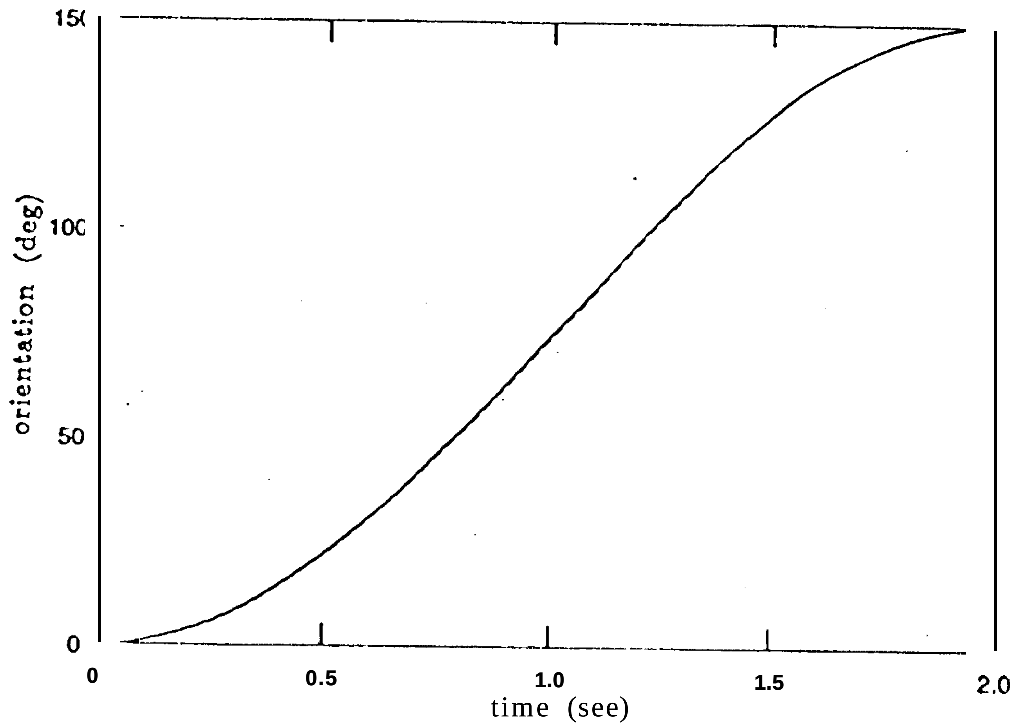

Figure 2c: Desired (dashed) and actual (solid) response of end-effecter orientation angle ϕ in PUMA arm simulation (nominal case)

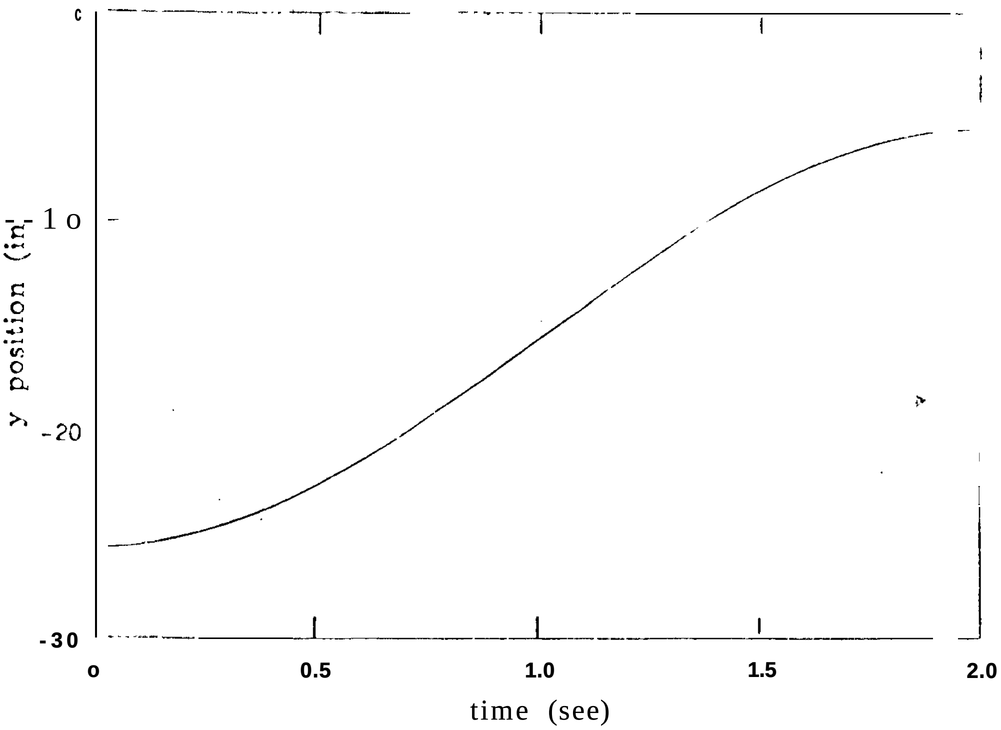

Figure 3a: Desired (dashed) and actual (solid) response of end-effecter y coordinate in PUMA arm simulation (time-varying payload)

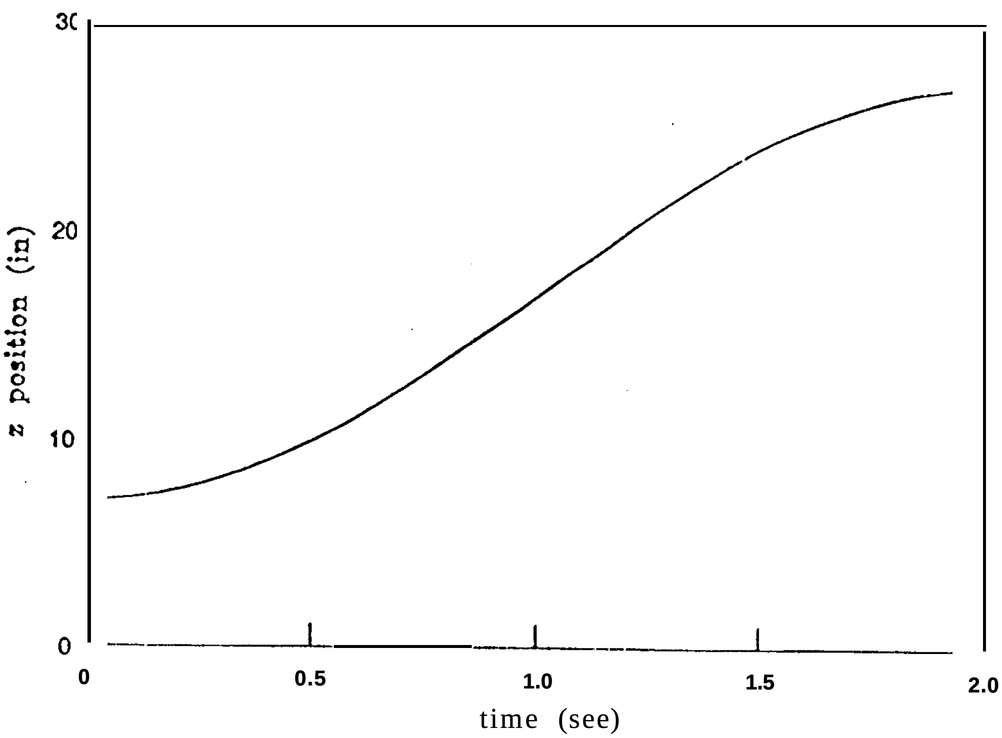

Figure 3b: Desired (dashed) and actual (solid) response of end-effector z coordinat e in PUMA arm simulation (time-varying payload)

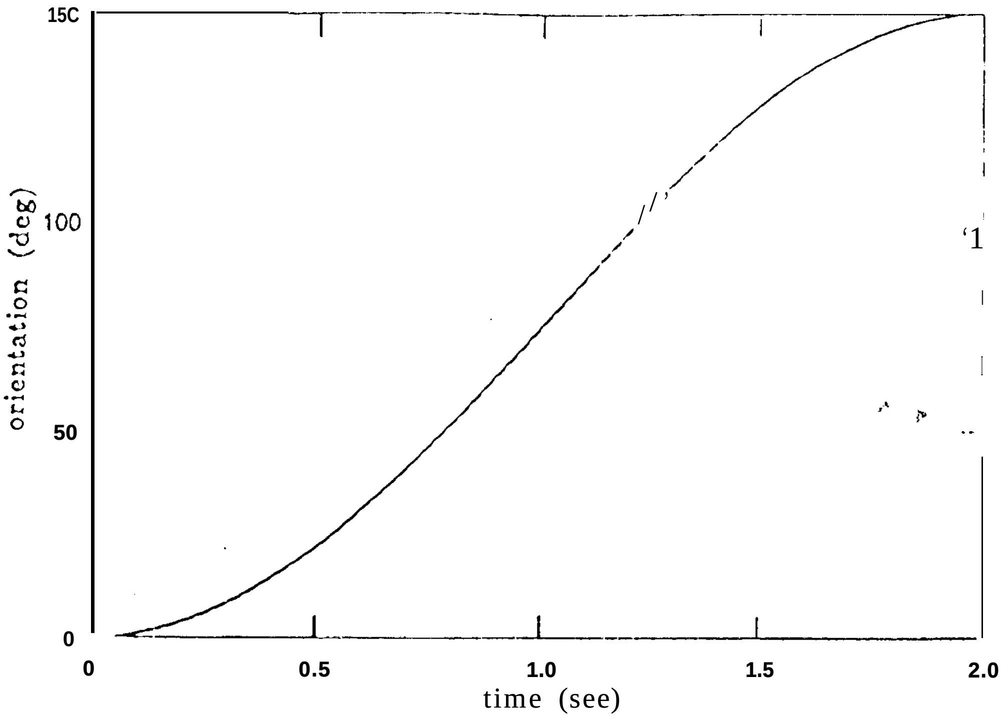

Figure 3c: Desired (dashed) and actual (solid) response of end-effecter orientation angle ϕ in PUMA arm simulation (time-varying payload)

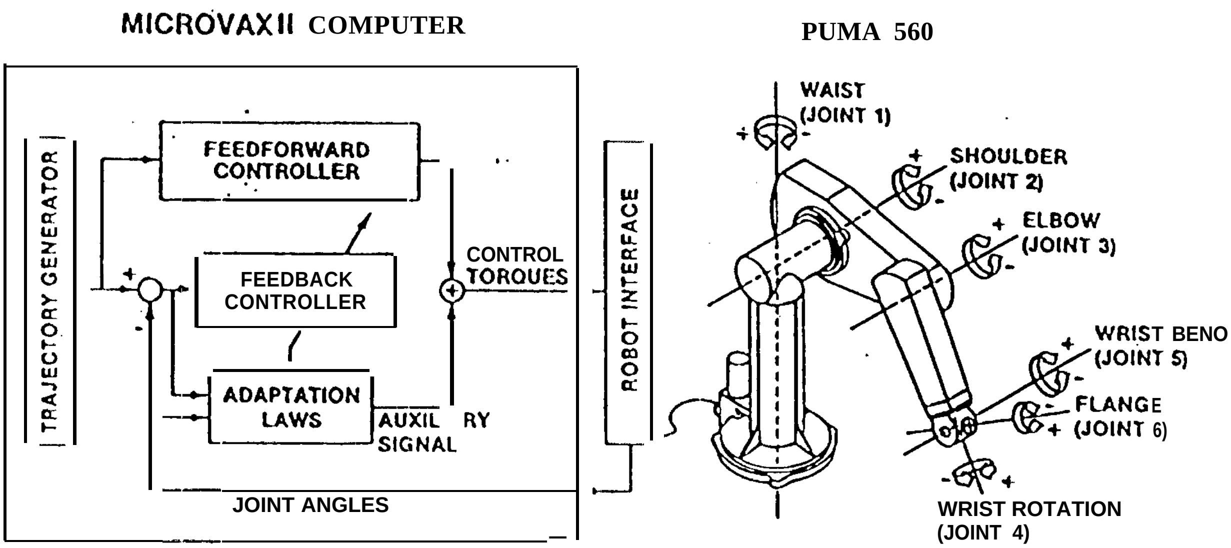

Figure 4: **Functional** diagram of testbed facility

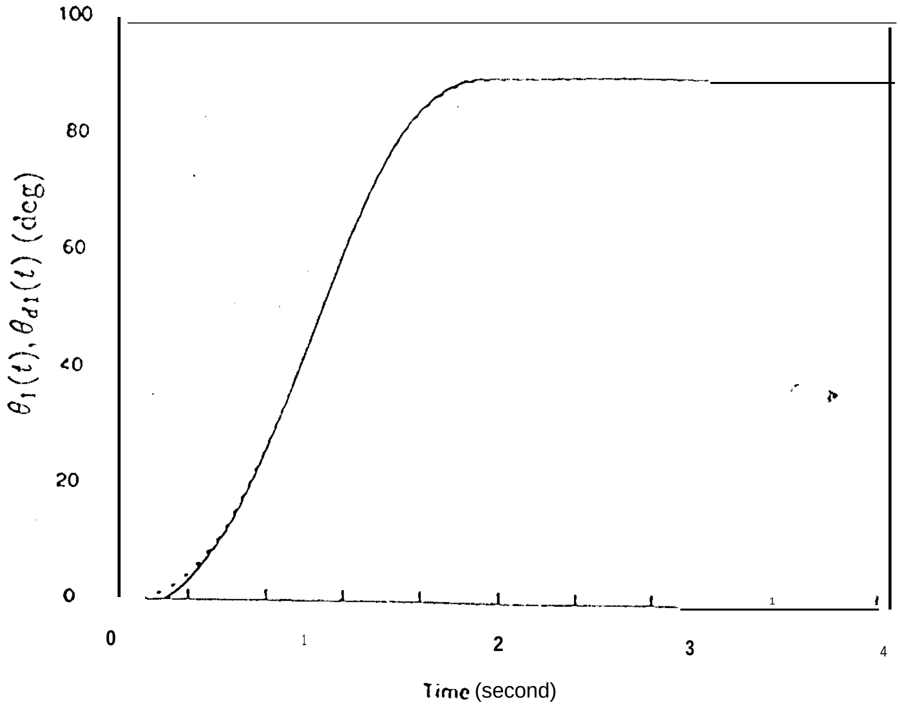

Figure 5a: Desired (dashed) and actual (solid) response of PUMA waist angle under adaptive controller

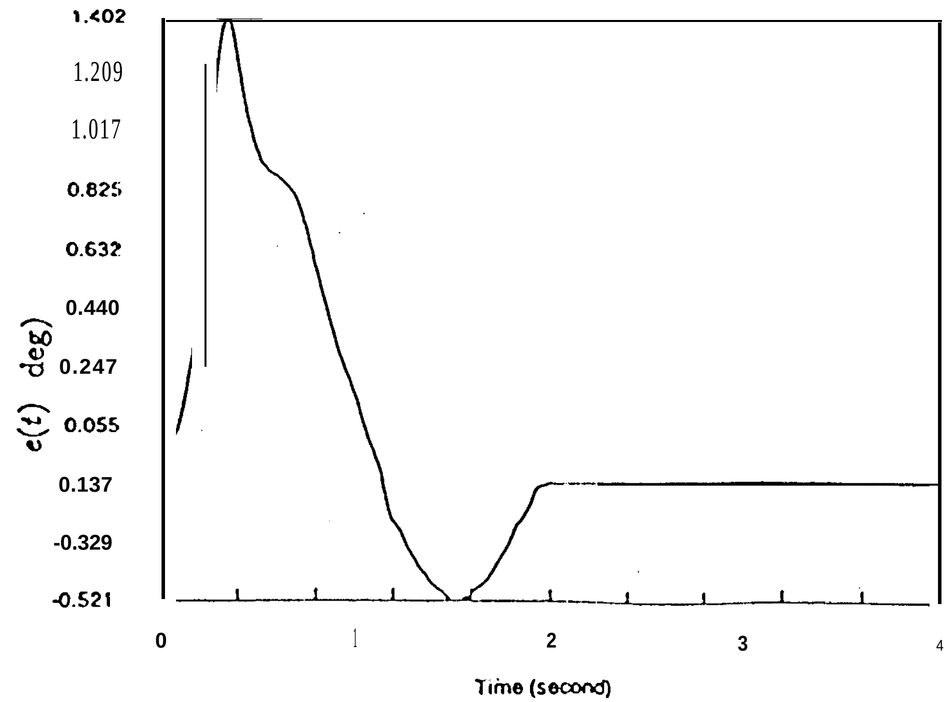

Figure 5b: Waist angle tracking error under adaptive controller

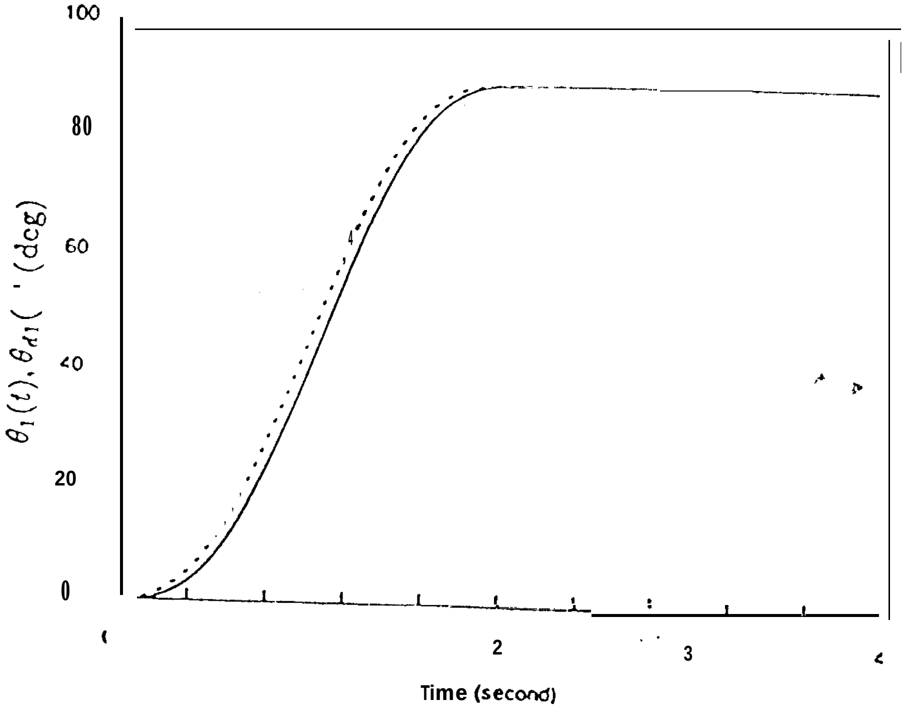

Figure 6a: Desired (dashed) and actual (solid) response of PUMA waist angle under Unimation controller

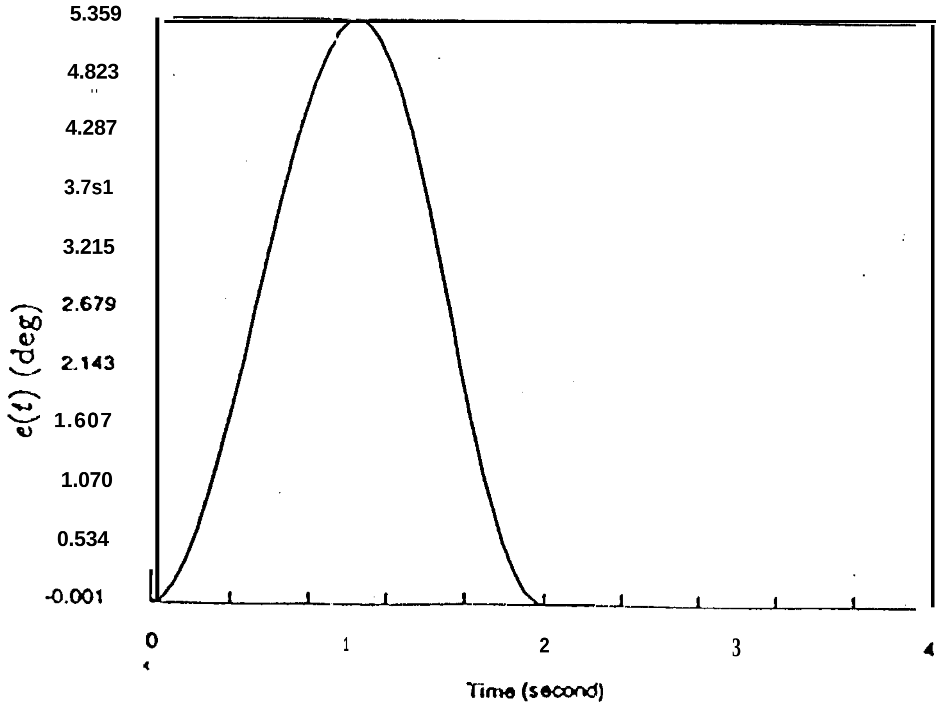

Figure 6b: Waist angle tracking error under Unimation controller

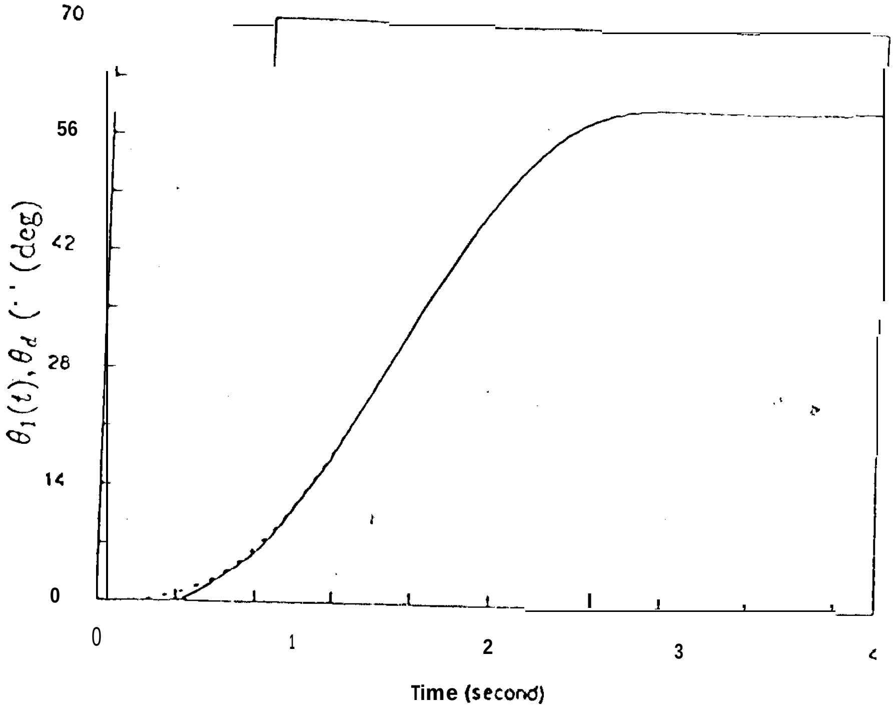

Figure 7a: Desired (dashed) and actual (solid) response of PUMA waist angle under adaptive controller with arm configuration change

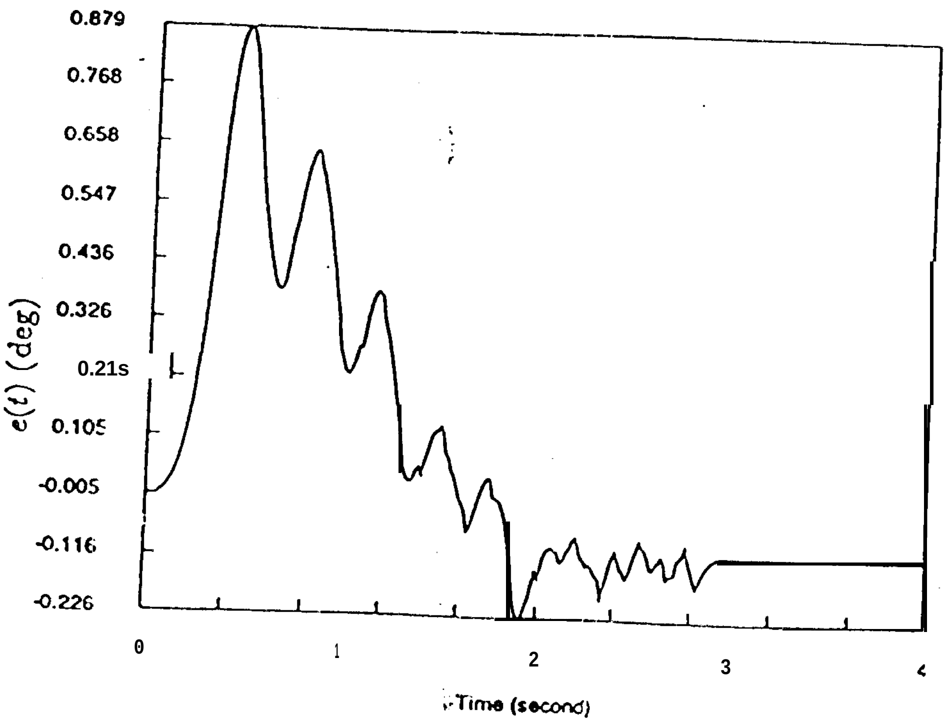

Figure 7b: Waist angle tracking error under adaptive controller with arm configuration change

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10. Appendix: Analysis of Error Bounds for Proposed Control Schemes

In this Appendix, we show that when there are no external disturbances acting on the